Approximate relations for the analysis of single-step stress-relaxation data in uniaxial extension from experiments involving a finite step time

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In the determination of single-step stress–relaxation behaviour, a finite time is required to reach the desired strain. As a result, an uncertainty is introduced into the observed behaviour at the early times. In the region of linear behaviour, an approximation has previously been derived which can be applied to shear stress–relaxation experiments. In the present work, approximate relations are derived which can be applied to uniaxial extension experiments in the region of non-linear behaviour. The derivations are based on the assumption that, under the set of strain histories considered, one can use the Bernstein, Kearsley and Zapas theory as a one-dimensional description. To demonstrate the validity of the approximate relations, we have obtained data on a linear low-density polyethylene copolymer under conditions of a varied step time and strains well into the region of non-linear behaviour.

(Keywords: BKZ theory; linear low-density polyethylene; mechanical behaviour; non-linear behaviour; stress relaxation; uniaxial extension)

INTRODUCTION

In single-step stress–relaxation experiments, a finite time is required during the application of the step to reach the desired strain. This introduces an uncertainty into the observed behaviour at the early times. Some years ago Zapas and Phillips[1] derived an approximation which can be applied to shear stress–relaxation experiments in the region of linear behaviour. Recently, Zapas, McKenna and Brenna[2] have derived corrections to the normal force response for the cone-and-plate geometry in single-step stress–relaxation experiments. For certain materials such as semicrystalline polymers, where some simplifying assumptions can be made, we have applied their idea to experiments involving simple extension at deformations which extend well into the region of non-linear behaviour. Approximate relations are derived which provide a correction to the behaviour observed at the early times. The derivations are based on the assumption that, for the type of strain histories considered here, one can use the Bernstein, Kearsley and Zapas (BKZ) theory[3] as a one-dimensional description. To show the validity of the approximate relations, we have obtained data on a linear low-density polyethylene copolymer at 26°C. The results are presented here. The theory shows that for this particular system one needs only one experiment in order to obtain a good approximation to the non-linear surface which describes the single-step stress–relaxation behaviour. This last statement must be qualified to the extent that, in practice, a larger set of experiments may be necessary if the experiments involve very large deformations.

DERIVATION OF SOME APPROXIMATE RELATIONS

We shall first consider an experiment in which the specimen, at time $t=0$, is subjected to a strain history in uniaxial extension involving a ramp function for a time t_1, followed by stress relaxation at constant strain.

The conditions are:

$$\begin{aligned} -\infty<\tau\leqslant 0 \qquad & \lambda(\tau)=1 \\ 0<\tau\leqslant t_1 \qquad & \lambda(\tau)=1+\kappa\tau \\ t_1<\tau\leqslant t \qquad & \lambda(\tau)=1+\kappa t_1=1+\varepsilon \end{aligned} \tag{1}$$

where $\lambda(\tau)$ is the stretch at time τ, $\lambda(\tau)=l(\tau)/l_0$, $l(\tau)$ and l_0 being respectively the deformed length at time τ and the initial undeformed length), κ is the rate of deformation and $\varepsilon(\tau)$ is the strain at time τ. We shall assume, for this particular strain history, that the BKZ theory[3] is applicable and that the true stress can be represented by the equation:

$$\sigma(t)=\kappa(\lambda,t)-\int_0^{t_1} K_*\left(\frac{\lambda}{1+\kappa\tau},\, t-\tau\right)\mathrm{d}\tau \tag{2}$$

$K(\lambda,t)$ is the value of the stress–relaxation function at time t after the introduction of a stretch λ, $K_*(\cdot,\cdot)$ is the partial derivative of $K(\lambda,t)$ with respect to the second argument, and $K(\lambda,\infty)$ and $K(1,t)$ are both zero. Equation (2) can be

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rewritten in the form:

$$\sigma(t)=\kappa\int_{t-t_1}^{t}K'\left(\frac{\lambda}{1+\kappa(t-\xi)},\xi\right)\left(\frac{\lambda}{[1+\kappa(t-\xi)]^2}\right)\mathrm{d}\xi \tag{3}$$

where $K'(\cdot,\cdot)$ now represents the partial derivative with respect to the first argument, and the variable change $t-\tau=\xi$ has been made.

By application of the first theorem of the mean for integrals, equation (3) can be expressed as:

$$\sigma(t)=C\kappa t_1K'\left(\frac{\lambda}{1+\kappa t_1/2},\, t-t_1/2\right)\left(\frac{\lambda}{(1+\kappa t_1/2)^2}\right) \tag{4a}$$

In the region where $t\geqslant t_1$:

$$\sigma(t)=C\varepsilon K'\left(\frac{\lambda}{1+\varepsilon/2},\, t-t_1/2\right)\left(\frac{\lambda}{(1+\varepsilon/2)^2}\right) \tag{4b}$$

In equations (4a) and (4b), the constant C represents a correction term to the mean value of the integral. If ε is small and if $K(\lambda,t)$ is expressed as

$$K\left[\frac{\lambda}{1+\varepsilon/2}+\frac{\lambda\varepsilon}{2+\varepsilon},\, t\right]$$

then $K(\lambda,t)$ can be expanded using a Taylor series expansion to obtain the relation:

$$K(\lambda,t)\simeq K\left(\frac{\lambda}{1+\varepsilon/2},\, t\right)+\left(\frac{\lambda\varepsilon}{2+\varepsilon}\right)K'\left(\frac{\lambda}{1+\varepsilon/2},\, t\right) \tag{5}$$

where terms in ε higher than first order have been dropped. Substitution of equation (5) into equation (4b) gives:

$$\sigma(t)=K(\lambda,t-t_1/2)A(\varepsilon,t-t_1/2) \tag{6}$$

where

$$A(\varepsilon,t-t_1/2)=\frac{2C}{1+\varepsilon/2}\left(1-\frac{K((1+\varepsilon)/(1+\varepsilon/2),\, t-t_1/2)}{K(1+\varepsilon,\, t-t_1/2)}\right)$$

For the case in which the material behaviour is linear and ε is small, $A(\varepsilon,t-t_1/2)$ approaches unity so that

$$\sigma(t)=K(\lambda,t-t_1/2)$$

which is equivalent to the relation obtained originally by Zapas and Phillips[1].

Now in order to calculate $A(\cdot,\cdot)$ it is necessary to determine the value of $K(\cdot,\cdot)$ at two different levels of strain, namely ε and $\varepsilon/2$. We shall next consider an experiment in which the following strain history applies:

$$-\infty<\tau\leqslant 0 \qquad \lambda(\tau)=1$$

$$0<\tau\leqslant t \qquad \lambda(\tau)=1+\kappa\tau$$

The assumption will now be made that the function $K(\lambda,t)$ can be represented by the relation:

$$K(\lambda,t)=\phi(\lambda)t^{-\alpha} \tag{7}$$

We shall see later that this assumption is justified for the system under consideration here since the stress–relaxation function can be represented on a log–log plot by a family of straight lines having the same slope α.

The equivalent of equation (2) then becomes:

$$\sigma(t)=K(\lambda,t)+\alpha\int_{0}^{t}\phi'\left(\frac{\lambda}{1+\kappa(t-\xi)}\right)\xi^{-1-\alpha}\,\mathrm{d}\xi \tag{8}$$

where $\phi'(\cdot)$ now represents the derivative of $\phi(\cdot)$ with respect to λ.

In the region where $K(\lambda,t)$, at constant t, is a monotonically increasing function of λ, the integral in equation (8) is of the order of 10% or less of the value of $K(\lambda,t)$. For small values of ε:

$$\phi\left(\frac{\lambda}{1+\kappa(t-\xi)}\right)\simeq\phi(1+\kappa\xi)$$

Inside the integral in equation (8), we shall further substitute for $\phi(1+\kappa\xi)$, the expression:

$$\phi(1+\kappa\xi)=\sigma(\xi)\xi^{\alpha} \tag{9}$$

where $\sigma(\xi)$ is the stress at any time ξ during the ramp portion of the deformation history. By substitution of equation (9) into equation (8) and once again the application of the theorem of the mean, the following expression is obtained:

$$\phi(1+\kappa t)=[\sigma(t)-2\alpha\sigma(t/2)]t^{\alpha} \tag{10}$$

$\phi(1+\kappa t)$ then represents to a very good approximation the stress–relaxation function at a time of 1 s, since

$$K(\kappa t,1)\simeq\phi(1+\kappa t)$$

With a knowledge of the function $\phi(\lambda)$, equation (3) can be rewritten as:

$$\sigma(t)=\kappa\lambda\int_{t-t_1}^{t}\phi'\left(\frac{\lambda}{1+\kappa(t-\xi)}\right)\left(\frac{\lambda}{[1+\kappa(t-\xi)]^2}\right)\mathrm{d}\xi \tag{11}$$

If we next make the substitutions $\theta=\xi/t_1$ and $R=t/t_1$, and divide both sides of equation (11) by

$$K(\lambda,t-t_1/2)=\phi(\lambda)(t-t_1/2)^{-\alpha}$$

then:

$$\frac{\sigma(t)}{K(\lambda,t-t_1/2)}=\frac{\varepsilon\lambda}{\phi(\lambda)}(R-\tfrac{1}{2})^{\alpha}\int_{R-1}^{R}\phi'\left(\frac{\lambda}{1+\varepsilon(R-\theta)}\right)\left(\frac{\theta^{-\alpha}}{[1+\varepsilon(R-\theta)]^2}\right)\mathrm{d}\theta \tag{12}$$

The right-hand side of equation (12) now depends only upon $R=t/t_1$ and ε, or

$$\sigma(t)/K(\lambda,t-t_1/2)=A(\varepsilon,R)$$

As an alternative approach to the more general treatment of the problem given above we can start directly from the condition that the strain ε is small, in which case the starting equation is then

$$\sigma(t)=\hat{K}(\varepsilon,t)-\int_0^{t_1}\hat{K}_*(\varepsilon(t)-\varepsilon(\tau),t-\tau)\mathrm{d}\tau \tag{13}$$

A caret has been placed above the function $K(\varepsilon,t)$ to distinguish it from $K(\lambda,t)$ used earlier. By applying the same procedures used to obtain equation (4a) the following result is obtained:

$$\sigma(t)=\kappa t_1\hat{K}'(\kappa t_1/2,t-t_1/2) \tag{14}$$

If we again assume that $\hat{K}(\varepsilon,t)$ can be represented as:

$$\hat{K}(\varepsilon,t)=\hat{\phi}(\varepsilon)t^{-\alpha} \tag{15}$$

then

$$\sigma(t)/\hat{K}(\varepsilon,t-t_1/2)=\varepsilon\hat{\phi}'(\varepsilon/2)/\hat{\phi}(\varepsilon) \tag{16}$$

where

$$\hat{\phi}'(\varepsilon/2)=\frac{2}{\varepsilon}\frac{\partial\ln\hat{\phi}(\varepsilon/2)}{\partial\ln(\varepsilon/2)}\frac{\hat{\phi}(\varepsilon/2)}{\hat{\phi}(\varepsilon)}$$

Next, the equivalent expression to equation (8) is:

$$\sigma(t)=\hat{K}(\varepsilon,t)+\alpha\int_0^t\phi(\kappa\xi)(\xi^{-1-\alpha})\mathrm{d}\xi \tag{17}$$

By substitution of the expression $\hat{\sigma}(\xi)=\hat{\phi}(\kappa\xi)\xi^{-\alpha}$ into equation (17) and a further application of the mean value theorem, the following expression for $\hat{\phi}(\varepsilon)$ results:

$$\hat{\phi}(\varepsilon)=[\sigma(t)-2\alpha\sigma(t/2)]t^{\alpha} \tag{18}$$

Equation (18) is equivalent to equation (10). Finally, we can rewrite equation (13) in the form:

$$\sigma(t)=\kappa\int_{t-t_1}^{t}K'(\kappa(t_1-t+\xi),\xi)\,\mathrm{d}\xi \tag{19}$$

Substitution of equation (15) into equation (19) then yields the relation:

$$\frac{\sigma(t)}{\hat{K}(\varepsilon,t-t_1/2)}=\frac{\varepsilon}{\hat{\phi}(\varepsilon)}(R-\tfrac{1}{2})^{\alpha}\int_{R-1}^{R}\hat{\phi}'(\varepsilon(1-R+\theta))\theta^{-\alpha}\,\mathrm{d}\theta \tag{20}$$

where $\theta=\xi/t_1$ and $R=t/t_1$. Equation (20) now substitutes for equation (12) where, as before, the right-hand side of the equation depends only on ε and R.

EXPERIMENTAL PROCEDURES

The polymer used in this study was an ethylene–hexene copolymer containing approximately 4.5 butyl branches per 1000 carbon atoms*. Its weight-average molecular weight was determined to be 170400 with a standard deviation of 4.5%[4]. As received, the polymer contained 0.075%, by weight, of a cadmium sulphoselenide pigment and a commercial stabilizer package. The density of the as-received pellets was $0.933\pm0.001\,\mathrm{g\,cm^{-3}}$ as determined using a density gradient column.

Tensile specimens were prepared by compression moulding the pellets in a heated press. The press was preheated to 175°C at which time the mould was placed in the press for 10 min under light contact pressure. After 10 min, the heat was turned off and the pressure was increased to 10 MPa (1450 psi). The press was then cooled under ambient conditions to a temperature below 70°C before the mould was removed from the press. Each specimen was then shaped to its final dimensions using a milling machine. The specimen dimensions were nominally 1.5 cm in width, 0.5 cm in thickness and 15 cm in length. The density of the moulded bars was determined, by hydrostatic weighing, to be $0.940\pm0.001\,\mathrm{g\,cm^{-3}}$.

Stress–relaxation experiments were done using a servo-controlled hydraulic test machine operated in strain control. The test machine was controlled by a computer which was programmed to output to the specimen a deformation consisting of a constant-rate-of-strain ramp function followed by a period of stress relaxation at constant strain. During the ramp portion of the experiment, the stress and strain were determined at intervals corresponding to 0.05 times the total ramp time t_1. During the stress–relaxation segment of the experiment, data were collected starting at the termination of the ramp function, 0.1 s later, and at intervals corresponding to each power of 2 in time thereafter. The duration of the stress–relaxation segment was either 102.4 or 819.2 s.

Three series of experiments were done in which the strain during the stress-relaxation portion of the experiment was 0.3%, 3% and 6%. In each series, the ramp time t_1 was varied from a time as short as 0.1 s to one as long as 100 s. It was found that at the smallest strain (0.3%) the same specimen could be used for the entire series of experiments. After the complete series of experiments, the first experiment was repeated, and no apparent difference in material behaviour was observed between the first and last test. We have also observed, in creep and recovery experiments done on the same material, that if the maximum strain during creep was kept below about 0.5% the creep and recovery behaviour could be described quite well using only a simple superposition principle. No 'plasticity' term was necessary, as was found to be the case for other polyethylenes[5–7].

At the two larger strains (3% and 6%), it was found necessary to use a fresh specimen for each experiment. This introduces the possibility of specimen-to-specimen variability which can influence the stress-relaxation behaviour. In order to obtain a measure of the possible specimen-to-specimen variability, a given experiment was repeated several times using a fresh specimen each time. In terms of isochronal data, it was found that there occurred a variability of about 10% from one specimen to

*This polyethylene is available in pellet form through the office of Standard Reference Materials, National Institute of Standards and Technology, Gaithersburg, Maryland 20899. Its designation is SRM 1497 another. Therefore, in some cases it was found necessary to repeat the same experiment until a series of experiments, at different times t_1, was obtained in which the stress-relaxation behaviour at the long times was in good agreement. At the longest times, the correction required as a result of the finite step time becomes negligible.

RESULTS AND DISCUSSION

In a single-step stress-relaxation experiment the material, which has been at rest at all times up to time $t=0$, is subjected to an 'instantaneous' step in strain, and the stress necessary to keep the specimen at that strain is measured as a function of time. In actual practice the strain cannot be applied in an instantaneous manner, but requires a finite time. For some semicrystalline polymers, particularly at relatively high levels of strain, it may be desirable to reach the predetermined strain via a strain history involving a rather slow ramp function, the purpose being to avoid possible heating or even fracture of the specimen. In simple extension, when the deformations are small enough and the behaviour can be described by linear viscoelasticity, one can use the approximation given by Zapas and Phillips[1]. This special case was discussed earlier in this paper where, to a very good approximation, $\sigma(t)=K(\lambda,t-t_1/2)$. In *Figure 1* we present data showing log σ(t) plotted *versus* $\log(t-t_1/2)$, obtained at various ramp rates, and where $\lambda=1.003$. The value of t_1 varied from 0.1 to 100 s. All of the data fall very close to those corresponding to the shortest ramp time (0.1 s), and are the correct values of the stress. In *Figure 2*, a similar plot is shown for the experiments done at stretches of 1.03 and 1.06. At these two much larger stretches, the curves approach one another only at times t greater than $5t_1$. It will be shown later that, for these values of λ, the behaviour is quite non-linear. It was shown earlier (equation (6)) that the relation $\sigma(t)=K(\lambda,t-t_1/2)A(\varepsilon,t-t_1/2)$ should describe the data quite well, where $A(\cdot,\cdot)$ is a correction factor which depends on ε and t/t_1. In general, $A(\cdot,\cdot)$ cannot be estimated without a reasonable knowledge of the non-linear surface of $K(\cdot,\cdot)$ for values of the stretch smaller than the final value of λ. For the case in which $K(\cdot,\cdot)$ can be represented as the product of a function of λ and a function of time of the type $\phi(\lambda)t^{-\alpha}$, the problem becomes manageable to the extent that one can obtain a reasonably good approximation for the function $\phi(\lambda)$ from the values of the stress during the ramp function. The derivation of this approximation was also given in an earlier section. The approximate value of the function $\phi(\lambda)$ was given by equation (10) as:

$$\phi(1+\kappa t)=[\sigma(t)-2\alpha\sigma(t/2)]t^{\alpha}$$

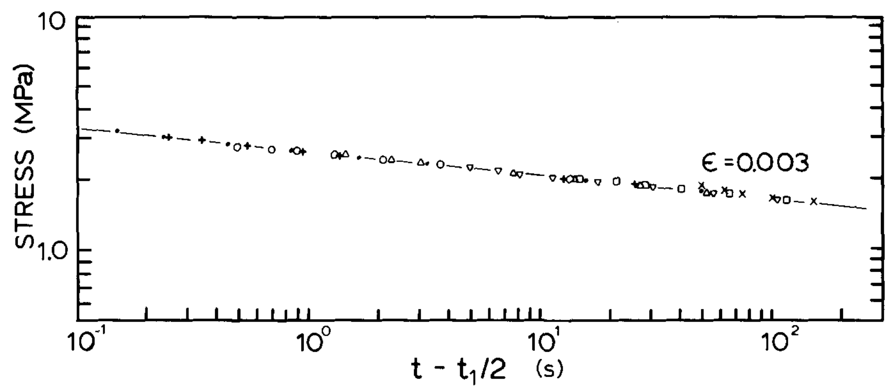

Figure 1 Log(stress) *versus* $\log(t-t_1/2)$ from stress-relaxation experiments. The step time t_1 was varied as follows: (●) 0.1 s; (+) 0.3 s; (○) 1.0 s; (△) 3.0 s; (▽) 10 s; (□) 30 s; and (×) 100 s

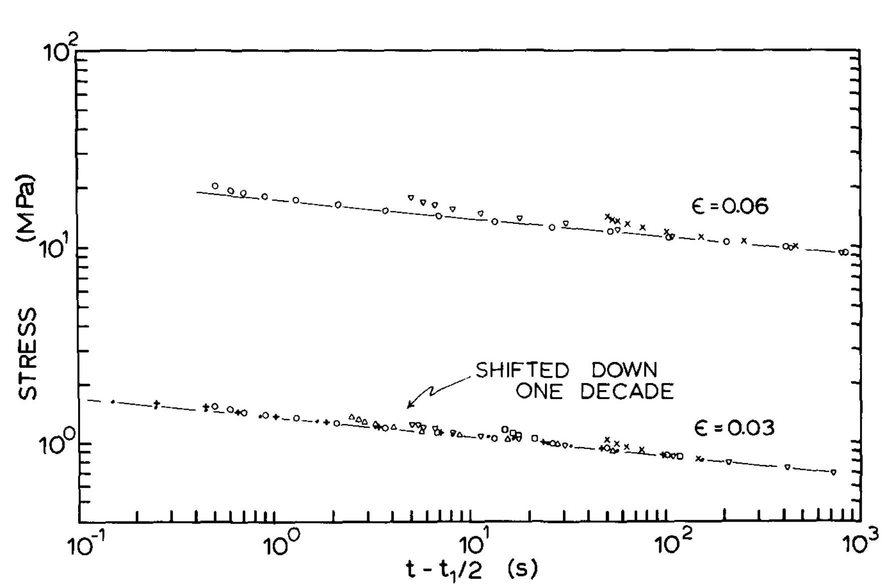

Figure 2 Log(stress) *versus* $\log(t-t_1/2)$ from stress relaxation. Symbols have the same meaning as in *Figure 1*

Here α represents the slope of the stress-relaxation curve and in our experiments was found to be -0.0987.

By having a good knowledge of the function $\phi(\lambda)$, one can calculate the function $A(\cdot,\cdot)$. In fact, for a given value of the function $\phi(\lambda)$, $A(\cdot,\cdot)$ can be normalized to obtain the form $A(\varepsilon, t/t_1)$. It was shown earlier that the ratio $\sigma(t)/K(\lambda, t-t_1/2)$ for values of t greater than t_1 was a function of $R=t/t_1$ and ε only. Since the time dependence of $K(\lambda,t)$ can be represented by a power-law function, it is easy to show that the ratio of $\sigma_1(R)$ to $\sigma_2(R)$ is a constant, where $\sigma_1(R)$ and $\sigma_2(R)$ are the stresses during the stress relaxation after ramp deformation histories at two different ramp rates to the same value of ε and with the same value of R. Several examples are shown in *Table 1* for the case where $\lambda=1.03$. From *Table 1*, it can be seen that the product of R^α and $\sigma_1(R)/\sigma_2(R)$ is in all cases unity. Similar results were also obtained for the case where $\lambda=1.06$. The calculated constant is the ratio of the two different times t_1 raised to the power α. By applying all of the above development, we were able to obtain the function $\phi(\lambda)$ for different values of the strain ε and the results are shown in *Figure 3*. The triangles represent the 1.0 s isochronal values from experiments in which t_1 was 0.1 s and the circles represent data calculated using equation (10). It can be seen that the agreement is indeed very good.

From equation (12), we were able to calculate the table of values for $A(\varepsilon,R)$ shown in *Table 2*. A maximum correction of about 10% occurs for the sample stretched to $\lambda=1.06$. $A(\varepsilon,R)$ approaches unity at smaller and smaller values of R as the stretch λ becomes smaller. For values of R greater than 5, $A(\varepsilon,R)$ is nearly unity. By applying the above corrections, we obtained the plot shown in *Figure 4*, where $\log[\sigma(t)/A(\varepsilon,R)]$ is plotted *versus* $\log(t-t_1/2)$. At the strain of $\varepsilon=0.03$ the points from the different experiments fall on the same straight line, at least within the range of times for which we have data. This type of behaviour was observed previously by Becker and Rademacher[8] who found that on log–log coordinates the stress-relaxation behaviour of

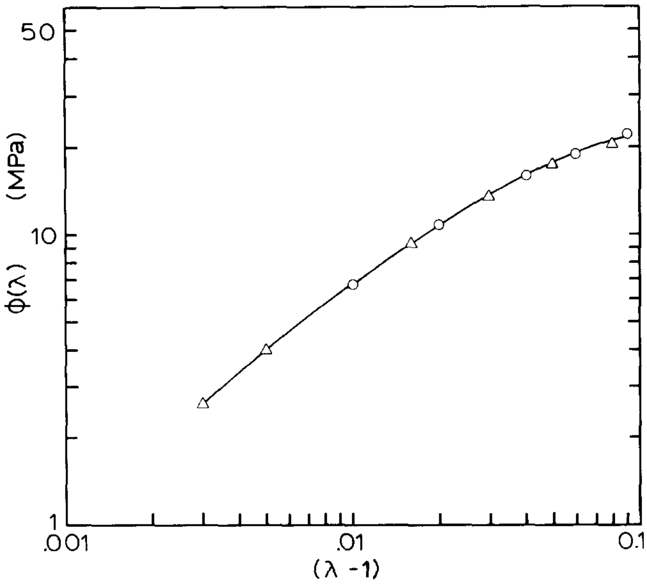

Figure 3 One second isochronal values of the function $\phi(\lambda)$ from stress-relaxation experiments in which the step time was 0.1 s. Triangles, values determined from experiments in which the stretch λ was varied from 1.003 to 1.08. Circles, values calculated using equation (10)

Table 2 Values of the correction factor $A(\varepsilon,R)^{a,b}$ as a function of R for different levels of strain

	Strain ε				
R	0.003	0.006	0.010	0.030	0.060
1.005	1.040	1.046	1.056	1.073	1.096
1.01	1.038	1.043	1.054	1.070	1.092
1.05	1.030	1.033	1.042	1.052	1.069
1.15	1.020	1.021	1.028	1.035	1.047
1.20	1.017	1.018	1.025	1.030	1.042
1.50	1.009	1.009	1.013	1.017	1.021
2.0	1.005	1.005	1.008	1.010	1.019
3.0	1.002	1.003	1.004	1.006	1.019
5.0	1.001	1.001	1.002	1.003	1.016
10.0	1.001	1.001	1.001	1.000	1.013

[a] Calculated using equation (12)
[b] $R=t/t_1$

Table 1 Table of values of $\sigma_{t_1}(R)$ and the ratio $\sigma_1(R)/\sigma_2(R)$ for a series of experiments in which $\lambda=1.03^{a-e}$

	$\sigma_{t_1}(R)$ (MPa)					$\sigma_1(R)/\sigma_2(R)$			
R	$t_1=1$	5	10	30	100	1/5	1/10	10/30	10/100
1.01			12.50	11.15	10.15			1.09	1.23
1.02		13.26	12.41	11.10	10.03			1.12	1.24
1.04		13.09	12.23	10.90	9.87			1.12	1.24
1.07		12.90	12.07	10.75	9.70			1.12	1.24
1.10	14.89	12.75	11.93	10.60	9.58	1.16	1.25	1.12	1.24
1.15	14.62	12.49	11.73	10.40	9.38	1.17	1.24	1.13	1.25
1.20	14.40	12.32	11.55	10.24	9.24	1.17	1.25	1.13	1.25
1.40	13.92	11.84	11.05	9.85	8.87	1.18	1.26	1.12	1.25
1.70	13.40	11.30	10.61	9.47	8.51	1.19	1.26	1.12	1.25
2.0	13.10	11.10	10.40	9.22		1.18	1.26	1.13	
2.5	12.68	10.70	10.00	8.91		1.18	1.27	1.12	
3.0	12.38	10.40	9.70	8.70		1.19	1.28	1.12	
4.0	11.19	10.00	9.40	8.46		1.12	1.27	1.11	
6.0	11.20	9.51	8.95	8.25		1.16	1.23	1.08	
			Average ratio			1.17	1.26	1.12	1.24
			$[\sigma_1(R)/\sigma_2(R)]R^\alpha$			1.00	1.01	1.01	0.99

[a] t_1 is the duration of the ramp deformation history
[b] $R=t/t_1$
[c] $\sigma_{t_1}(R)$ is the stress during the stress-relaxation portion of the experiment after a ramp deformation history of duration t_1
[d] $\sigma_1(R)/\sigma_2(R)$ is the ratio of two stresses obtained from different experiments in which the strain ε and R were the same, but the time t_1 was different as indicated by the column headings
[e] $\alpha=-0.0987$ (slope of the stress-relaxation curve plotted on log–log paper)

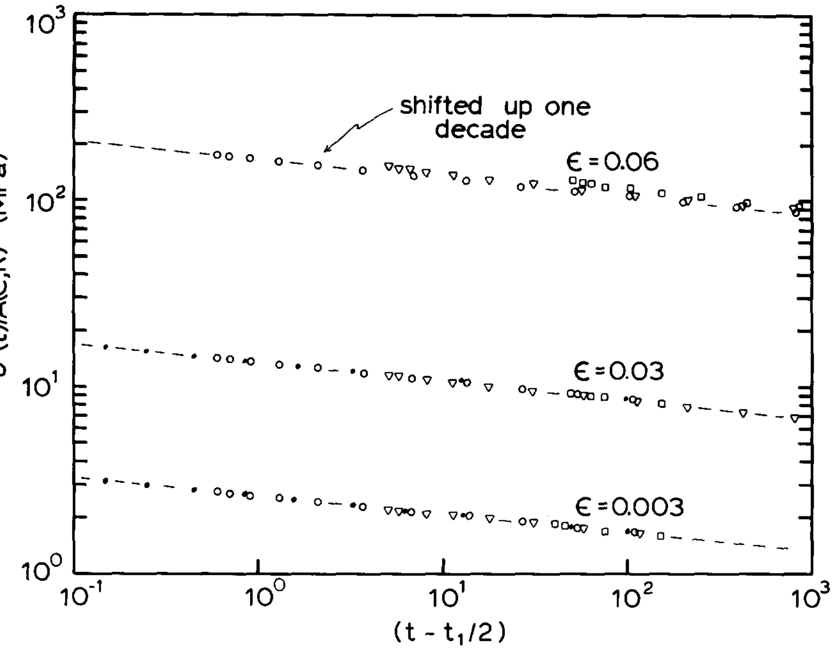

Figure 4 Log[$\sigma(t)/A(\varepsilon,R)$] *versus* log($t-t_1/2$). $A(\varepsilon,R)$ was calculated using equation (12)

polyethylenes having widely different crystallinities could be represented by a similar power-law function.

At the largest strain ($\varepsilon = 0.06$) the values do not all quite fall on the line corresponding to 1 s. The departure appears to become greater the longer the step time t_1. It is possible that sample-to-sample variability may account for some of the differences, in which case the various values will not fall on the same line even at very long times. However, it is more likely that in the form in which they were used the approximate relations undercorrect the values at very large strains and cannot fully account for the non-linearity in the system behaviour. We do know that at very large strains the BKZ theory does not fully describe the behaviour observed in materials of this type.

While a step time of 100 s may appear unrealistic, the usefulness of the procedure which we have just outlined can be demonstrated in the following situation where we attempted to obtain the stress-relaxation behaviour of a specimen extended to a stretch of 1.12. We were successful only when the value of t_1 was greater than 30 s. For a ramp time of 30 s or shorter the specimen necked and fractured before reaching the desired stretch. For some classes of polymeric materials it may be desirable to use a relatively slow step time in order to achieve even relatively small strains.

In summary, a set of approximate relations is developed for the analysis of single-step stress-relaxation data obtained in uniaxial extension and involving a finite step time. The derivations are based on the assumption that, for the set of strain histories considered, the predictions of the BKZ theory are applicable. We have demonstrated that the relations, while approximate, provide a very good description of the material response even well into the region of non-linear behaviour. The procedures outlined should be particularly useful in instances where it may be desirable to reach the predetermined strain via a slower ramp function, the purpose being to avoid possible rapid heating or even fracture of the specimen during the application of the step in strain.

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